**THE INVERSE TRIGONOMETRIC FUNCTIONS**

Consider the trigonometric functions and their inverse relationships below:

**y = sin(x) and x = sin(y)**



**y = cos(x)** and **x = cos(y)**



**y = tan(x)** and **x = tan(y)**



The inverse relation can be inverse functions if the domain is restricted,

It is conventional to define the inverse functions as follows:

Domain  and range [-1, 1] Domain [-1, 1] and range 

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| |  |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | Domain  and range (-∞, ∞) Domain (-∞, ∞) and range  **Summary**   |  |  |  | | --- | --- | --- | | Function | Domain | Range | | y = sin-1(x) = arc sin(x) | [-1, 1] |  | | y = cos-1(x) = arc cos(x) | [-1, 1] |  | | y = tan-1(x) = arc tan(x) | (-∞, ∞) |  |   **How to differentiate the inverse trig functions**:  Given y = sin-1(x) then x = sin (y). = ? | | Alternatively, we can consider the triangle where sin (y) = x        The third side is  so if .  Likewise given y = cos-1(x) then x = cos (y). ?    x = cos (y)  OR using the triangle        If  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_  Given y = tan-1(x) then x = tan (y). ?    OR        If | |  |  |  |

**SUMMARY**

|  |  |
| --- | --- |
| Function | Derivative |
| y = sin-1(x) |  |
| y = cos-1(x) |  |
| y = tan-1(x) |  |

**Also**

|  |  |
| --- | --- |
| Function | Derivative |
| y = sin-1f(x) |  |
| y = cos-1f(x) |  |
| y = tan-1f(x) |  |

**Examples**

Find the derivative of each of the following:

(a) y = tan-1(x)



(b) y = tan-1(5x)



(c) y = tan-1(4+x)

 OR 

(d) y = arcsin(x2 – 3)



In general

If y = arc sin(xn) then 

**Also**

|  |  |
| --- | --- |
| Function | Derivative |
| y = sin-1f(x) |  |
| y = cos-1f(x) |  |
| y = tan-1f(x) |  |

More examples:

(e) y = cos-1(3x-2)



(f) y = tan-1()



(g) y = sin-1(4x2)



(h) y = x2cos-1(x) Using the product rule:



(i) If f(x) = sin-1x + cos-1x, show that f’(x) = 0



Mental work: State the derivatives of

  

**THE INTEGRATION OF TRIGONOMETRIC FUNCTIONS.**

Given

|  |  |
| --- | --- |
| Function | Derivative |
| y = sin-1(x) |  |
| y = cos-1(x) |  |
| y = tan-1(x) |  |

|  |
| --- |
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|  |

It follows that

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|  |
|  |

**and**

**likewise**

**Examples**

(a)  Experiment with the substitution u = 2x

(b) 

(c) 

(d) 

(e) 

(f) 

(g) 

(h) 

(i) 

(j) 

(k) 

(l) 

(m) 

(n) 

Sometimes the transformation required before we integrate is a little more complicated.

For example



The rule illustrated here is 

Prove this rule:



Examples







1. Show that

(a) 

(b) 

(c) 

2. Use the substitution  to show .

3. Show that

(a) 

(b) 

(c) 